EXTENSION OF SVD-DFT ANALYSIS FOR A CLASS OF NON-LINEAR SYSTEMS

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Abstract: The paper develops computer algebra based method and tools for a class of non-linear time-varying (LTV), discrete-time systems. Proposed method base on two following transformations: from general NL system into state-space dependent piecewise linear (PWL) and from PWL into LTV. LTV system is then decomposed using method based on Singular Value Decomposition, Discrete Fourier Transform and Power Spectral Density into approximated Bode diagrams. Potential applications are simplified analysis and synthesis in frequency domain of weak and/or slow nonlinear systems. Especially the synthesis can be done for each time instant which pretend the method for applications in predictive control. The paper begins from short literature review, description of the model, detailed algorithm with description of the proposed method and two numerical examples.

Keywords: Non-linear systems, Discrete-time systems, Time-varying systems, Frequency methods.

1. INTRODUCTION

Frequency methods are one of the most important tools for linear, time-invariant (LTI) systems analysis, nevertheless well-developed concepts and analytic methods of time-invariant systems cannot be applied, even to small class of non-linear (NL) systems.

Employed approach base on two following transformations: from general NL system into state-space dependent piecewise linear (affine) (PWL,PWA) and from PWL into LTV. Then, approximated LTV system is decomposed using SVD-DFT method worked out by (Orlowski 2004) into approximated Bode diagrams. The method take advantage of two mentioned above concepts extensively applied in model predictive control. See for instance: state space dependent form (Dutka, Ordys 2004, Ordys, Clarke 1993) and PWL system (Bacic, Cannon, Kouvaritakis 2003, Grancharova, Johansen, Tondel 2005).

Frequency approach has been actively employed for NL systems in last decades. Many works concerned on the analysis on influence of non-linearity in the system on input-output spectra of the system. Papers are often concerned on NL systems driven by periodic multiharmonic signals (Chua 1979, Schetzen 1980). As the result NL distortions has been classificated into harmonic and interharmonic contributions (Billings 1989, Solomou 2002). Results of the analysis are often exploit for the system identification (Evans 1994, Schoukens 1998, Pintelon 2001).

The purpose of this work is to propose the method for simplified NL systems analysis using approximated Bode diagrams. The reasoning base on the concepts taken from LTV systems. The first time-varying transfer function has been defined by extending the Laplace transform to the varying impulsive response by Zadeh (1950). Later works of frequency aspects for LTV systems focuses on modal analysis. Ideas of varying eigenvalues or varying natural frequencies have been used without a rigorous definition e.g. by Maia (1997). Other concept of pseudo-modal parameters *PMP* was described e.g. by Liu (1999). The pseudo-modal parameters are related to the eigenvalues of the varying discrete-time state transition matrices by analogy to time-invariant systems.

The *SVD-DFT* analysis for LTV systems gives not only natural frequencies, but also Bode diagrams (amplitude and phase). Nevertheless Bode diagrams are given by the finite set of frequencies (or singular vectors) and corresponding amplifications. The products of *SVD-DFT* analysis are the characteristics (amplitude and phase). It results in that, the information included in the diagrams cannot be extracted for specific time samples. An important advantage of *SVD-DFT* method is that, the characteristics calculated for LTI systems are almost identical as like classical Bode diagrams. It is important that the simple physical interpretation of Bode diagrams (e.g. amplitude magnification and phase shifting) is true only for LTI systems. For LTV and NL systems there exists approximation which holds up only some properties of the classical diagrams. It should be also noted that frequency methods can be applied not to large class of nonlinear systems.

Our main motivation is to propose a simple method that can be used for NL time varying (TV) communication channels including mobile communication (Strohmer 2005) and for slow NL systems. The system is understand as slow NL when the working point is changes slowly and not rapidly, e.g. when nonlinearity cause only in slow transitions between different working points. As it was mentioned earlier proposed method take advantage of double transformation (NL-PWL-LTV) which can be made for each time instant. Bode diagrams may be estimated also for predicted variables. In such case the approximated diagrams can be used for simplified calculus of control in MPC or adaptative methods using simplified frequency domain approach.

2. NONLINEAR MODEL

In general a non-linear, discrete-time control system is described by following equation

$$\mathbf{x}_p(k+1) = \mathbf{f}_0(\mathbf{x}_p(k), \mathbf{v}_p(k), k), \qquad \mathbf{x}_p(0)=\mathbf{x}_0, \quad (1)$$

In such case it is very difficult to carry out the analysis. Much more specific model for non-linear systems is described below. In fact it is state space model with non-linear coefficients, called state space dependent form.

$$\mathbf{x}_p(k+1) = \mathbf{A}(\mathbf{x}_p(k), k)\cdot \mathbf{x}_p(k) + \mathbf{B}(\mathbf{v}_p(k), k)\cdot \mathbf{v}_p(k) \quad (2)$$

$$\mathbf{y}_p(k) = \mathbf{C}(\mathbf{x}_p(k), k)\cdot \mathbf{x}_p(k), \quad \mathbf{x}_p(0)=\mathbf{x}_0,\ k\in\mathbf{N}, \quad (3)$$

where $\mathbf{x}_p(\cdot)\in(\mathbf{R}^n)^N$ is the state, $\mathbf{v}_p(\cdot)\in(\mathbf{R}^m)^N$ the control, $\mathbf{y}_p(\cdot)\in(\mathbf{R}^p)^N$ is the output, and $\mathbf{A}(\mathbf{x}_p(k), k)\in\mathcal{L}(\mathbf{R}^n)$, $\mathbf{B}(\mathbf{v}_p(k), k)\in\mathcal{L}(\mathbf{R}^m, \mathbf{R}^n)$, $\mathbf{C}(\mathbf{x}_p(k), k)\in\mathcal{L}(\mathbf{R}^n, \mathbf{R}^p)$ are known arrays of functions.

Making the generalisation of frequency analysis for NL systems it is necessary to know the characteristic behaviour of the system. For LTI systems full information is included e.g. in the coefficients of transfer function. For LTV systems full information is included for example in the system operator. Such operator can be further decomposed into eigenvectors of the system.

For the NL systems full information is included into the model of the system, but how can one isolate characteristic behaviour of the system ?

A. First possibility is to determine eigenvectors of the systems employing knowledge of the model and own experience. The eigenvectors have to be orthonormal basis. For linear systems there exists only one orthonormal basis, however for NL systems there can exist much more bases (even infinity number of bases). Each new eigenvector non-orthogonal to previous eigenvectors make a new basis. Thus transforming the bases using DFT it is possible to compute frequency characteristics. Non-linearity can be interpreted as tendency to change bases.

B. Second possibility is conversion of one NL system into *n* LTV systems. Accuracy obtained for such transformation is much more higher then for simple linearisation to one LTI model. Choice of suitable number of models *n*, input signals (IS) and initial conditions (IC) require some knowledge about the system. The input variables (IS and IC) should ensure that the conversion from NL into *n*-LTV will be representative for this system for various input functions. In such case the eigenvectors can be done using singular value decomposition (SVD). Below it will be shown the procedure to determine eigenvectros using the B method.

The first step in conversion from NL to *n*-LTV system is to replace non-linear matrix functions dependent on state, input and time $\mathbf{A}(\mathbf{x}_p(k), k)$, $\mathbf{B}(\mathbf{v}_p(k), k)$, $\mathbf{C}(\mathbf{x}_p(k), k)$ by time-varying matrices dependent only on time $\mathbf{A}(k), \mathbf{B}(k), \mathbf{C}(k)$. This step shold be repeated *n* times for *n* representative input functions. Next steps have to be repeated for all *n* models. Any obtained model can be described by following model

$$\mathbf{x}_p(k+1) = \mathbf{A}(k)\cdot \mathbf{x}_p(k) + \mathbf{B}(k)\cdot \mathbf{v}_p(k), \quad (4)$$

$$\mathbf{y}_p(k) = \mathbf{C}(k)\cdot \mathbf{x}_p(k), \qquad k\in\mathbf{N}, \quad \mathbf{x}_p(0)=\mathbf{0}, \ (5)$$

where $\mathbf{x}_p(\cdot)\in(\mathbf{R}^n)^N$ is nominal state, $\mathbf{v}_p(\cdot)\in(\mathbf{R}^m)^N$ is nominal control, $\mathbf{y}_p(\cdot)\in(\mathbf{R}^p)^N$ is nominal output, and $\mathbf{A}(k)\in\mathcal{L}(\mathbf{R}^n)$, $\mathbf{B}(k)\in\mathcal{L}(\mathbf{R}^m, \mathbf{R}^n)$, $\mathbf{C}(k)\in\mathcal{L}(\mathbf{R}^n, \mathbf{R}^p)$ are system matrices.

Alternatively, discrete-time system converted to LTV can be given as sum or matrix operator. Matrix operator's notation is given by

$$\hat{\mathbf{L}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{A}(1) & \mathbf{I} & \mathbf{0} & \vdots & \vdots \\ \vdots & \ddots & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{A}(N-2)\cdot\ldots\cdot\mathbf{A}(1) & \cdots & \mathbf{A}(N-2) & \mathbf{I} & \mathbf{0} \end{bmatrix} \quad (6)$$

$$\hat{\mathbf{N}} = \begin{bmatrix} \mathbf{I} \\ \mathbf{A}(0) \\ \vdots \\ \mathbf{A}(N-2)\cdot\ldots\cdot\mathbf{A}(0) \end{bmatrix} \quad (7)$$

matrix operators $\hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$ have diagonal form i.e.

$$\hat{\mathbf{B}} = \begin{bmatrix} \mathbf{B}(0) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}(N-1) \end{bmatrix} \quad \hat{\mathbf{C}} = \begin{bmatrix} \mathbf{C}(0) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}(N-1) \end{bmatrix} \quad (8)$$

and vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{v}}$ have following notation

$$\hat{\mathbf{x}} = \begin{bmatrix} \mathbf{x}_\mathbf{p}(0) \\ \vdots \\ \mathbf{x}_\mathbf{p}(N-1) \end{bmatrix}, \ \hat{\mathbf{y}} = \begin{bmatrix} \mathbf{y}_\mathbf{p}(0) \\ \vdots \\ \mathbf{y}_\mathbf{p}(N-1) \end{bmatrix}, \ \hat{\mathbf{v}} = \begin{bmatrix} \mathbf{v}_\mathbf{p}(0) \\ \vdots \\ \mathbf{v}_\mathbf{p}(N-1) \end{bmatrix} \quad (9)$$

Output trajectory of the system can be given in the following form

$$\hat{\mathbf{y}} = \hat{\mathbf{C}}\hat{\mathbf{L}}\hat{\mathbf{B}}\cdot\hat{\mathbf{v}} + \hat{\mathbf{C}}\hat{\mathbf{N}}\cdot\mathbf{x}_0 \quad (10)$$

The operator $\hat{\mathbf{C}}\hat{\mathbf{L}}\hat{\mathbf{B}}$ for LTV system is a compact and Hilbert-Schmidt operator from l_2 into l_2 and actually maps boundedly signals $\mathbf{v}(k) \in \mathcal{U} = l_2[0, N]$ into signals $y \in \mathcal{Y}$.

The SVD of system operator can be done using described above matrix operators. Such decomposition presents a generalization of the classic SVD of matrices (Golub 1983). This is possible because operators defined for discrete-time systems over a finite time horizon are finite dimensional. For such systems the time horizon is product of sampling period of the system and total number of samples.

SVD in linear algebra decomposes the operator into corresponding sets of singular values σ_i, singular input vectors $\mathbf{v}_i$ and singular output vectors $\mathbf{u}_i$. Any complex or real matrix $\mathbf{X}$ may be written as a product of three matrices $\mathbf{X} = \mathbf{U}\cdot\Sigma\cdot\mathbf{V}^*$, where $\Sigma = \mathrm{diag}(\sigma_i)$ is a diagonal matrix, and orthonormal matrices $\mathbf{U}$, $\mathbf{V}$ are composed of column vectors $\mathbf{u_i}$ and $\mathbf{v_i}$ respectively.

Response $\mathbf{y}_v = \sigma_i \cdot \mathbf{u}_i$ of a singular value decomposed $\mathbf{U}\cdot\mathbf{S}\cdot\mathbf{V}^\mathrm{T} = \hat{\mathbf{C}}\cdot\hat{\mathbf{L}}\cdot\hat{\mathbf{B}}$ LTV system having been excited by the input $\hat{\mathbf{v}} = \mathbf{v}_i$ defined by the i-th column of the $\mathbf{V}$ matrix, is equal to the product of i-th singular value and i-th column of the $\mathbf{U}$ matrix.

To start the next stage of analysis one must decompose the system as sets of matrices **U, S, V**. Matrices **U, V** should be orthonormal matrices containing eigenvectors output and input, respectively. Matrix **S** contain corresponding magnifications for particular eigenvectors. Such sets are result of SVD analysis and are obtained automatically doing procedure B. For method A such sets have to be obtained manually. Independent on the method (A or B) further procedure is identical.

3. THE TRANSFORM THEOREMS

Theorem 1 Discrete power density spectrum of every orthogonal matrix computed as a sum of spectral density column vectors is constant and equal to **1**.

In particular, for matrix $\mathbf{V}=\{\mathrm{v_{ij}}\}$, i,j=1…$N$,

$$\begin{aligned} \mathbf{S}_\mathrm{v}(\omega_k) &= \sum_{j=1}^{N} S_j(\omega_k) = \frac{1}{N}\cdot\sum_{j=1}^{N}\left|\mathrm{DFT}_k[\mathbf{v}_j]\right|^2 \\ &= \frac{1}{N}\cdot\sum_{i=1}^{N}\left|\sum_{n=1}^{N} v_{ni}\cdot e^{-j\cdot 2\cdot\pi\cdot(k-1)\cdot(n-1)/N}\right|^2 = \mathbf{1} \end{aligned} \quad (11)$$

where $\omega_k = \dfrac{k}{2\cdot T_p\cdot N}$, T_p – sampling period.

Proof of the Theorem follows directly from the orthonormality of the SVD matrix (Golub 1983) and from unitary properties of the DFT transform. The following equation holds true then

$$\left|\mathrm{DFT}_k\left[\mathbf{v}_j\right]\right|^2 = \mathbf{1} \quad (12)$$

hence

$$S(\omega_k) = \frac{1}{N}\cdot\sum^{N} 1 = 1 \quad (13)$$

Thus the theorem is proved.

Theorem 2. Discrete input-output power density spectrum of system, can be computed as a sum of spectral density column vectors of product **U·S**.

The notation is following

$$\begin{aligned} \mathbf{S}_\mathrm{y}(\omega_k) &= \frac{1}{N}\cdot\sum_{j=1}^{N}\left|\mathrm{DFT}_\mathrm{k}[\mathbf{u}_j\cdot s_{jj}]\right|^2 \\ &= \frac{1}{N}\cdot\sum_{i=1}^{N}\left|\sum_{n=1}^{N} u_{ni}\cdot\sigma_i\cdot e^{-j\cdot 2\cdot\pi\cdot(k-1)\cdot(n-1)/N}\right|^2 \end{aligned} \quad (14)$$

where $\omega_k = \dfrac{k}{2 \cdot T_p \cdot N}$, T_p – sampling period, $\sigma_i = s_{ii}$ – i^{th} – singular value of the decomposition $\mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}^{\mathrm{T}} = \hat{\mathbf{C}}\hat{\mathbf{L}}\hat{\mathbf{B}}$.

Proof of the theorem follows directly from SVD properties, especially from orthonormality of **U**, **V** matrices.

4. AMPLITUDE AND PHASE CHARACTERISTICS APPROXIMATION

The relation between input and output power spectrum density and amplitude characteristics are described following.

$$\mathbf{S}_{\mathbf{y}}(\omega_k) = \left|\mathbf{G}(\omega_k)\right|^2 \cdot \mathbf{S}_{\mathbf{x}}(\omega_k) \tag{15}$$

taking into account theorem 1,

$$\left|\mathbf{G}(\omega_k)\right| = \sqrt{\mathbf{S}_{\mathbf{y}}(\omega_k)} \tag{16}$$

and finally

$$\begin{aligned}\left|\mathbf{G}(\omega_k)\right| &= \sqrt{\frac{1}{N} \cdot \sum_{j=1}^{N} \sigma_j^2 \cdot \left|\mathrm{DFT}_{\mathrm{k}}[\mathbf{u}_j]\right|^2} \\ &= \sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N} \left|\sigma_i \cdot \sum_{n=1}^{N} u_{ni} \cdot e^{-j \cdot 2 \cdot \pi \cdot (k-1) \cdot (n-1)/N}\right|^2}\end{aligned} \tag{17}$$

Phase characteristics, can be approximated by

$$\begin{aligned}\varphi(\omega_k) &= \arg\left(\sum_{j=1}^{N} \sigma_j \cdot \frac{\mathrm{DFT}_{\mathrm{k}}[\mathbf{u}_j]}{\mathrm{DFT}_{\mathrm{k}}[\mathbf{v}_j]}\right) \\ &= \arg\left(\sum_{i=1}^{N}\left(\sigma_i \cdot \frac{\sum_{n=1}^{N} u_{ni} \cdot e^{-j \cdot 2 \cdot \pi \cdot (k-1) \cdot (n-1)/N}}{\sum_{n=1}^{N} v_{ni} \cdot e^{-j \cdot 2 \cdot \pi \cdot (k-1) \cdot (n-1)/N}}\right)\right)\end{aligned} \tag{18}$$

Operator's notation, for which has been defined fundamental frequency analysis tools, is useful for description and simulation for time variant and time invariant systems and for a class of nonlinear systems all of these systems are defined on finite time horizon.

Developed algorithm enable computing one pair of diagrams: amplitude and phase for each set of decomposed matrices **U, S, V**. For *n*-LTV system the result of analysis will be family of *n* diagrams pairs. Representation of NL system can be also understood as area on the magnitude diagram.

5. NUMERICAL EXAMPLES

Selected results of frequency characteristics approximation for two non-linear systems using discrete operators, SVD and DFT for three different systems are presented below.

5.1 Quadratic, strong non-linear system

Let us consider following non-linear discrete-time system

$$\begin{aligned} x_{k+1} &= x_k^2 + u_k^3 \\ y_k &= x_k \end{aligned} \tag{19}$$

Properties of the system are strongly dependent on the level of the state and input signal. For small signals, the output goes fast to zero, for signals near but less unity output tends slow to zero and for signals above unity the system is unstable.

Let us assume that expected level of state will be in the range $x \in < 0.01, 0.8 >$.

The system have strong connection between dynamics of the system and the level of the state. It is assumed that input signals will have amplitudes in the range <0.01,0.8>. As the test signals have been chosen sinusoidal inputs with angular frequency of 3 and 10 rad/s. Magnitude and phase frequency responses are depicted on fig. 1. Corresponding time responses are plotted on fig. 2. It may be concluded from fig. 1 that the area of non-linear transitions (NLT) of amplitude diagram is large. Expanding the range amplitudes for input signals the NLT area will widen to full plane (towards -∞ for signals with levels tends to zero and towards +∞ for levels above 1).

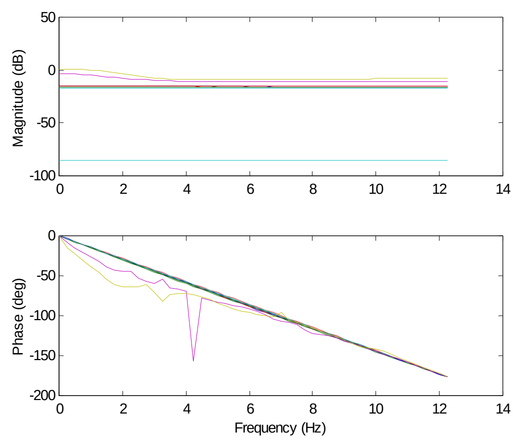

Figure 1. Amplitude and phase diagrams for quadratic system $x \in < 0.01, 0.8 >$, 10 simulations, horizon N=100 steps.

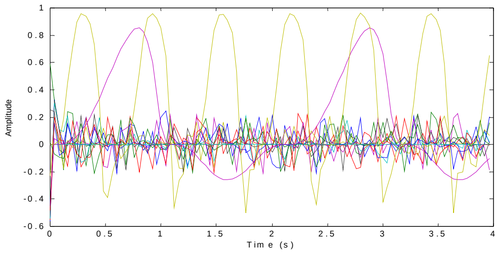

Figure 2. Output responses for quadratic systems (sinusoidal excitation).

The results of frequency analysis are unsatisfactory and the developed tools should not be used for such or similar systems. Moreover the SVD of such systems for state signals' levels $x \rightarrow 0$ or $x \geq 1$ is numerically instable.

5.2 Oscillatory, weak non-linear system

The system is discretised analogue oscillatory element with non-linear coefficients. The system can be described by following differential equation

$$\ddot{y}(t) + \beta(|\dot{y}(t)|) \cdot \omega_0(y(t)) \cdot \dot{y}(t) + \omega_0^2(y(t)) \cdot y(t) = \omega_0^2(y(t)) \cdot u(t) \tag{20}$$

The non-linear coefficients β, ω_0 are given in the polynomial form.

$$\beta(t) = b_2 \cdot (\dot{y}(t))^2 + b_1 \cdot |\dot{y}(t)| + b_0$$
$$\omega_0(t) = w_2 \cdot (y(t))^3 + w_2 \cdot (y(t))^2 + w_1 \cdot y(t) + w_0 \tag{21}$$

It is assumed that the coefficients of the polynomials are following:

$$\mathbf{b} = [b_2, b_1, b_0] = [0.001,\ 0,\ 0.001]$$
$$\mathbf{w} = [w_3, w_2, w_1, w_0] = [0.002,\ 0,\ 0.02,\ 4] \tag{22}$$

Characteristics of the coefficients β, ω_0 vs. $\dot{y}$ and y respectively are plotted on fig. 3.

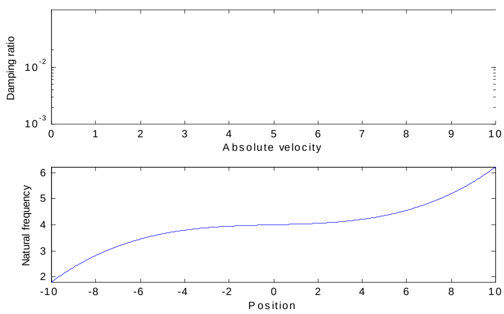

Figure 3. Coefficients β (damping ration), ω_0 (natural frequency) vs. generalised position and absolute velocity (y and $\dot{y}$ respectively).

First step in the analysis is to convert the model to state space and discretise it. Corresponding, matrices of discretised model (2-3) can be written as follows:

$$\mathbf{A}(\mathbf{x}(k), k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -\beta(x_1(k)) \cdot \omega_0(x_2(k)) & -\omega_0^2(x_2(k)) \\ 1 & 0 \end{bmatrix} \cdot T_p \tag{23}$$
$$\mathbf{B}(\mathbf{u}(k), k) = \begin{bmatrix} 1 & 0 \end{bmatrix}^{\mathrm{T}} \cdot T_p$$
$$\mathbf{C}(\mathbf{x}(k), k) = \begin{bmatrix} 0 & \omega_0^2(x_2(k)) \end{bmatrix}$$

where: x_1 is generalised velocity
x_2 is generalised position
T_p is sampling period

It is assumed that the levels of state variables are less than 10 ($|x_1| < 10, |x_2| < 10$). As the test inputs have been chosen 400 random signals with rectangular distribution <-8,8> and zero initial conditions. Magnitude and phase frequency responses are depicted on fig. 4. NLT area of amplitude characteristics is bounded. The phase characteristics is indeterminate for approx. f>1Hz.

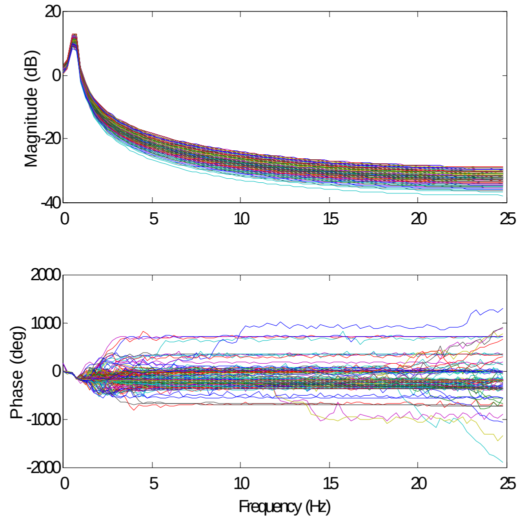

Figure 4. Amplitude and phase diagrams for oscillatory system $u \in <-8, 8>$, 400 simulations, horizon N=200 steps.

Similar results can be obtained for other input signals, e.g. sinus, step functions and nonzero initial conditions, under assumption of boundedness of state variables to given interval. Lesser number of simulations follows of course lower accuracy for the NLT area.

Plotted diagrams allow to select appropriate controller for closed feedback loop, similarly as for linear systems. Moreover NCT area of amplitude diagram can be used as a measure of the degree of nonlinearity of the system. Set of the approximated Bode diagrams allow to make equivalent linear, uncertain, noised model for the system. Splitting of the phase diagram may be converted to uncertainty while indeterminacy of the diagram for larger frequencies may be converted to additional input with white noise passed by high pass filter. Such system may be easily synthesized using well known methods for linear systems, e.g Robust Control Toolbox.

6. CONCLUSION

Developed method can be used for weak non-linear systems, for which frequency diagrams make sense. For analysis strong non-linear systems, like quadratic system described in section 5.1 one have to use another methods. Exact definite classes of systems for which the method can be used is difficult.

The main advantages of the developed concepts are analogies to classical frequency methods for LTI systems. Presented method can be useful tool for preliminary or simplified analysis for non-linear systems. Possible future extension is estimating the degree of non-linearity of the system. For example the measure can be the NLT area of amplitude diagrams.

The method could be used either for time-invariant or time-varying systems, but always on finite time horizon. If the non-linearity and variability existing in the system are neglected, the computed diagrams are in assumption the same as for corresponding LTI system. The main advantage of the *SVD-DFT* method is existence of extension for analysis for time-varying systems. It is difficult to compare results for LTV and NL systems, because classical Fourier Transform was not defined for such a systems.

Newertheless, the results for practical applications look optimistically. In spite of many advantages of the method, there exist a few weak points, which were appeared during simulations e.g. numerical instability for some signals. There exists wide spectrum of possible applications, systems analysis and estimating, not only for time varying communication channels but also for determining the degree of system non-stationarity and non-linearity.

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