AN INTRODUCTION TO SVD-DFT FREQUENCY ANALYSIS FOR TIME-VARYING SYSTEMS

PRZEMYSŁAW ORŁOWSKI

Technical University of Szczecin, Control Engineering Institute,
Sikorskiego 37, 70-313 Szczecin, e-mail: orzel@ps.pl

Abstract. The paper develops tools and methods for linear time-varying, discrete-time systems analysis. It consists of theoretical background, definitions and numerical algorithms for frequency characteristics approximation. The main method is based on Singular Value Decomposition *SVD*, Discrete Fourier Transform *DFT* and power density spectrum properties. A few illustrative numerical examples are included. Three different models have been analysed: oscillatory element, low pass filter and variable structure system. For better evaluation, results for presented method are compared with classical Bode characteristics.

Keywords. Discrete-time systems, Time-varying systems, Non-stationary systems.

1. INTRODUCTION

Frequency methods are one of the most important tools for linear, time-invariant (LTI) systems analysis, nevertheless well-developed concepts and analytic methods of time-invariant systems cannot be applied to linear time-varying systems (LTV). As well the study of stability and dynamics problems are different on finite time horizon. The analysis for a set with infinite length is used essentially a frequency domain description. The tool, which is used to describe a system by its frequency-dependent amplification, is quite useful, but use for plants considered on a finite time interval is not known. This work presents tools for time-varying systems using approximation of classical, frequency methods.

One of the first attempts for analysis LTV systems in the frequency domain has been made by Zadeh (1950). The time-varying transfer function has been defined by extending the Laplace transform to the varying impulsive response. However, in general, no closed-form of the Zadeh's transfer function is known. Recent works of frequency aspects for LTV systems focuses on modal analysis. Ideas of varying eigenvalues or varying natural frequencies have been used without a rigorous definition e.g. by Maia (1997). The concept of pseudo-modal parameters *PMP* was introduced and described by Liu (1999). The pseudo-modal parameters are related to the eigenvalues of the varying discrete-time state transition matrices by analogy to time-invariant systems.

An analysis of *SVD-DFT* gives not only natural frequencies, but also Bode characteristics (amplitude and phase). Nevertheless Bode characteristics are given by the finite set of frequencies (or singular vectors) and corresponding amplifications. The physical properties of the system are dependent not only on the poles but also on the zeros and on the gain of the system, which are neglected using *PMP*. The products of *SVD-DFT* analysis are the characteristics (amplitude and phase). It results in that, the information included in characteristics cannot be extracted for specific time samples. Extracting the exact nature of the varying sometimes is easier using *PMP*. An important advantage of *SVD-DFT* method is that, the characteristics calculated for LTI systems are almost identically as like classical Bode's diagrams. Moreover, as the result of analysis, the varying degree coefficient of the system is defined and computed. If the set of the system matrices have been taken by system identification, the coefficient gives an information, whether the system is LTI, LTV or non-linear.

2. MODEL DESCRIPTION

Dynamic, discrete-time system can be given by set of difference equations, called the state space model

$$\mathbf{x}_p(k+1) = \mathbf{A}(k)\cdot\mathbf{x}_p(k) + \mathbf{B}(k)\cdot\mathbf{v}_p(k), \qquad (1)$$

$$\mathbf{y}_p(k) = \mathbf{C}(k)\cdot\mathbf{x}_p(k), \qquad k\in\mathbf{N},\ \mathbf{x}_p(0)=\mathbf{0}, \quad (2)$$

where $\mathbf{x}_p(\cdot)\in(\mathbf{R}^n)^N$ is nominal state, $\mathbf{v}_p(\cdot)\in(\mathbf{R}^m)^N$ is nominal control, $\mathbf{y}_p(\cdot)\in(\mathbf{R}^p)^N$ is nominal output, and $\mathbf{A}(k)\in\mathcal{L}(\mathbf{R}^n)$, $\mathbf{B}(k)\in\mathcal{L}(\mathbf{R}^m,\mathbf{R}^n)$, $\mathbf{C}(k)\in\mathcal{L}(\mathbf{R}^n,\mathbf{R}^p)$ are system's matrices.

Alternatively, the system can be given as sum or matrix operator. Sum operator is given by

$$\mathbf{y}_p(k) = \left(\hat{\mathbf{C}}\cdot\mathbf{L}^{\mathrm{F}}\cdot\hat{\mathbf{B}}\left[\mathbf{v}_p(\cdot)\right]\right)(k) = \mathbf{C}(k)\cdot\left(\sum_{i=0}^{k-2}\left[\prod_{j=i+1}^{k-1}\mathbf{A}(j)\right]\cdot\mathbf{B}(i)\cdot\mathbf{v}_p(i) + \mathbf{B}(k-1)\cdot\mathbf{v}_p(k-1)\right) \qquad (3)$$

The operator $\hat{\mathbf{C}}\cdot\mathbf{L}^{\mathrm{F}}\cdot\hat{\mathbf{B}}$ is a compact and Hilbert-Schmidt operator from l_2 into l_2 and actually maps boundedly signals $\mathbf{u}(k)\in\mathcal{U}=l_2[0,N]$ into signals $y\in\mathcal{Y}$.

Matrix operator is given by

$$\mathbf{L}^{\mathbf{F}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{L}^{\mathbf{P}}(1,1) & \mathbf{I} & \mathbf{0} & \vdots & \vdots \\ \vdots & \ddots & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{L}^{\mathbf{P}}(1,N-2) & \cdots & \mathbf{L}^{\mathbf{P}}(N-2,N-2) & \mathbf{I} & \mathbf{0} \end{bmatrix} \qquad (4)$$

$$\mathbf{N}^{\mathbf{F}} = \begin{bmatrix} \mathbf{I} \\ (\mathbf{N}^{\mathbf{F}})(1) \\ \vdots \\ (\mathbf{N}^{\mathbf{F}})(N-1) \end{bmatrix} \qquad (5)$$

where

$$\mathbf{L}^{\mathbf{P}}(i,k) = \prod_{j=i}^{k}\mathbf{A}(j) \qquad (\mathbf{N}^{\mathrm{F}}\mathbf{z})(k) = \prod_{j=0}^{k-1}\mathbf{A}(j)\cdot\mathbf{z} \qquad (6)$$

and matrix operators $\hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$ have diagonal form i.e.

$$\hat{\mathbf{B}} = \begin{bmatrix} \mathbf{B}(0) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}(N-1) \end{bmatrix} \quad \hat{\mathbf{C}} = \begin{bmatrix} \mathbf{C}(0) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}(N-1) \end{bmatrix} \qquad (7)$$

where vectors $\mathbf{x_p}(\cdot)$, $\mathbf{y_p}(\cdot)$ and $\mathbf{v_p}(\cdot)$ have following notation

$$\mathbf{x_p}(\cdot) = \begin{bmatrix} \mathbf{x_p}(0) \\ \vdots \\ \mathbf{x_p}(N-1) \end{bmatrix} \quad \mathbf{y_p}(\cdot) = \begin{bmatrix} \mathbf{y_p}(0) \\ \vdots \\ \mathbf{y_p}(N-1) \end{bmatrix} \quad \mathbf{v_p}(\cdot) = \begin{bmatrix} \mathbf{v_p}(0) \\ \vdots \\ \mathbf{v_p}(N-1) \end{bmatrix} \qquad (8)$$

3. DISTRIBUTION THEOREMS

The method is based on Singular Value Decomposition of the system operator. This spectral decomposition is a generalisation for SVD of a matrix. For discrete-time systems and finite time horizon the operator is finite dimensional. In linear algebra, the SVD of a matrix describes it by a set of singular values σ_i and corresponding sets of singular input-vectors v_i and output vectors u_i. Any real or complex matrix X can be written as $X=U\cdot\Sigma\cdot V^*$, where $\Sigma=\mathrm{diag}\{\sigma_i\}$ and U and V are composed from u_i and v_i, respectively.

Theorem 1 For every operators $\hat{\mathbf{C}}$, $\mathbf{L}^{\mathrm{F}}$, $\hat{\mathbf{B}}$, which define dynamical system on finite time horizon and matrices **U, S, V**, products of singular value decomposition

$$\mathbf{U}\cdot\mathbf{S}\cdot\mathbf{V}^{\mathrm{T}} = \hat{\mathbf{C}}\cdot\mathbf{L}^{\mathrm{F}}\cdot\hat{\mathbf{B}} \qquad (9)$$

the energy amplification for single input, single output systems (SISO), defined as output to input energy ratio at zero initial conditions, belong to following interval

$$\sigma_{\min}(\hat{\mathbf{C}}\cdot\mathbf{L}^{\mathrm{F}}\cdot\hat{\mathbf{B}}) \le k_e \le \sigma_{\max}(\hat{\mathbf{C}}\cdot\mathbf{L}^{\mathrm{F}}\cdot\hat{\mathbf{B}}) \qquad (10)$$

where

$$k_e = \sqrt{\frac{\sum_{i=1}^{N} y^2(i)}{\sum_{i=1}^{N} v^2(i)}} = \sqrt{\frac{\mathbf{y}^{\mathrm{T}}\cdot\mathbf{y}}{\mathbf{v}^{\mathrm{T}}\cdot\mathbf{v}}} \qquad (11)$$

is the energy amplification at $\mathbf{x_0}=\mathbf{0}$.

Proof:

$$\sum_{i=1}^{N} y^2(i) = \mathbf{y}^{\mathrm{T}}\cdot\mathbf{y} = \mathbf{v}^{\mathrm{T}}\cdot\mathbf{V}\cdot\mathbf{S}^2\cdot\mathbf{V}^{\mathrm{T}}\cdot\mathbf{v}$$

Matrix **V** is orthonormal base in $\mathbf{R}^N$, **S** is diagonal matrix, hence scalar product belong to interval (10).

Conclusion: H_2 norm of the system $\hat{\mathbf{C}}\cdot\mathbf{L}^{\mathrm{F}}\cdot\hat{\mathbf{B}}$ can be computed as

$$\left\|\hat{\mathbf{C}}\cdot\mathbf{L}^{\mathrm{F}}\cdot\hat{\mathbf{B}}\right\|_2=\sup_{\mathbf{v}} k_e=\sigma_{\max}(\hat{\mathbf{C}}\cdot\mathbf{L}^{\mathrm{F}}\cdot\hat{\mathbf{B}}) \qquad (12)$$

Theorem 2 For every operators $\hat{\mathbf{C}}$, $\mathbf{N}^{\mathrm{F}}$, which define dynamical system on finite time horizon and matrices **U, S, V**, products of singular value decomposition. The energetical capacity for SISO systems, defined as output to initial energy ratio at zero input energy, belong to following interval

$$\sigma_{\min}(\hat{\mathbf{C}}\cdot\mathbf{N}^{\mathrm{F}})\le c_e\le\sigma_{\max}(\hat{\mathbf{C}}\cdot\mathbf{N}^{\mathrm{F}}) \qquad (13)$$

where

$$c_e=\sqrt{\frac{\sum_{i=1}^{N} y^2(i)}{\sum_{j=1}^{n} x_0^2(j)}}=\sqrt{\frac{\mathbf{y}^{\mathrm{T}}\cdot\mathbf{y}}{\mathbf{x_0}^{\mathrm{T}}\cdot\mathbf{x_0}}} \qquad (14)$$

define the energetical capacity at $\mathbf{v}(\cdot)\equiv\mathbf{0}$.

Proof is analogical as for previous theorem.

Theorem 3 For every SISO system, and their singular value decompsition products $\mathbf{U}\cdot\mathbf{S}\cdot\mathbf{V}^{\mathrm{T}}=\hat{\mathbf{C}}\cdot\mathbf{L}^{\mathrm{F}}\cdot\hat{\mathbf{B}}$, when input are a sequence of i-column of matrix **V** and zero initial conditions, the output response is a product of i-singular value and i-column of matrix **U**.

Theorem 4 For every SISO system, and their singular value decompsition products $\mathbf{U}\cdot\mathbf{S}\cdot\mathbf{V}^{\mathrm{T}}=\hat{\mathbf{C}}\cdot\mathbf{N}^{\mathrm{F}}$, when input is zero and the initial conditions are a sequence of i-column of matrix **V**, the output, initial response is a product of i-singular value and i-column of matrix **U**.

Proofs of theorems **3-4** follow directly from the orthonormality of matrices **U**, **V** and properties of the *SVD*.

Control sequence $\mathbf{v_p}(\cdot)$ or initial conditions $\mathbf{x_0}$, for which energy amplification or energetical capacity is maximal are given by the first column of matrix **V**. Corresponding minimal values are given by the last column of matrix **V**.

4. THE TRANSFORM THEOREMS

Theorem 5 Discrete power density spectrum of every orthogonal matrix computed as a sum of spectral density column vectors is constant and equal to **1**.

In particular, for matrix **V**=$\{v_{ij}\}$, i,j=1…*N*,

$$\mathbf{S}_{\mathbf{v}}(\omega_k)=\sum_{j=1}^{N}S_j(\omega_k)=\frac{1}{N}\cdot\sum_{j=1}^{N}\left|\mathrm{DFT}_k[\mathbf{v}_j]\right|^2$$
$$=\frac{1}{N}\cdot\sum_{i=1}^{N}\left|\sum_{n=1}^{N}v_{ni}\cdot e^{-j\cdot2\cdot\pi\cdot(k-1)\cdot(n-1)/N}\right|^2=\mathbf{1} \qquad (15)$$

where $\omega_k=\dfrac{k}{2\cdot T_p\cdot N}$, T_p – sampling period.

Proof:

$$\mathbf{S}(\omega_k)=\frac{1}{N}\cdot\sum_{i=1}^{N}\left(\sum_{n=1}^{N}v_{ni}\cdot e^{-j\cdot2\cdot\pi\cdot(k-1)\cdot(n-1)/N}\right)\cdot\left(\sum_{n=1}^{N}v_{ni}\cdot e^{-j\cdot2\cdot\pi\cdot(k-1)\cdot(n-1)/N}\right)^*$$

replacing a_{ni} by Discrete Fourier Transform

$$a_{ni}=v_{ni}\cdot e^{-j\cdot2\cdot\pi\cdot(k-1)\cdot(n-1)/N} \qquad (16)$$

for every *k*=1…*N,* there are

$$\mathbf{S}(\omega_k)=\frac{1}{N}\cdot\sum_{i=1}\left(a_{1i}+a_{2i}+\ldots+a_{Ni}\right)\cdot\left(a_{1i}^*+a_{2i}^*+\ldots+a_{Ni}^*\right)$$
$$=\frac{1}{N}\cdot\sum_{i=1}^{N}\begin{pmatrix}a_{1i}\cdot a_{1i}^*+a_{1i}\cdot a_{2i}^*+\ldots+a_{1i}\cdot a_{Ni}^*+\\ a_{2i}\cdot a_{1i}^*+a_{2i}\cdot a_{2i}^*+\ldots+a_{2i}\cdot a_{Ni}^*+\\ \vdots\quad\vdots\quad\vdots\quad\vdots\\ a_{Ni}\cdot a_{1i}^*+a_{Ni}\cdot a_{2i}^*+\ldots+a_{Ni}\cdot a_{Ni}^*\end{pmatrix}$$
$$=\frac{1}{N}\cdot\sum_{i=1}^{N}\left[\sum_{n=1}^{N}a_{ni}\cdot a_{ni}^*+\sum_{m=1}^{N-1}\sum_{n=1}^{N-m}\left(a_{ni}\cdot a_{(n+m),i}^*+a_{(n+m),i}\cdot a_{ni}^*\right)\right]$$

using (16) it is easy to rearrange the equation

$$a_{ni}\cdot a_{(n+m),i}^*+a_{(n+m),i}\cdot a_{ni}^*=v_{ni}\cdot v_{(n+m),i}\cdot\left(e^{-j\cdot2\cdot\pi\cdot(k-1)\cdot m/N}+e^{j\cdot2\cdot\pi\cdot(k-1)\cdot m/N}\right)$$
$$=v_{ni}\cdot v_{(n+m),i}\cdot2\cdot\cos(m\cdot2\cdot\pi\cdot(k-1)/N) \qquad (17)$$

and

$$a_{ni}\cdot a_{ni}^*=v_{ni}^2 \qquad (18)$$

holding the orthogonality there are

$$\sum_{i=1}^{N}\left(a_{ni}\cdot a_{(n+m),i}^*+a_{(n+m),i}\cdot a_{ni}^*\right)=0\quad\text{for every m}\neq0$$

$$\sum_{n=1}^{N}a_{ni}\cdot a_{ni}^*=1$$

hence

$$\mathbf{S}(\omega_k) = \frac{1}{N} \cdot \sum_{i=1}^{N} 1 = 1$$

Which finish proof of (15).

Theorem 6. Discrete input-output power density spectrum of system, can be computed as a sum of spectral density column vectors of product **U·S**.

The notation is following

$$\begin{aligned} \mathbf{S}_{\mathrm{y}}(\omega_k) &= \frac{1}{N} \cdot \sum_{j=1}^{N} \left| \mathrm{DFT}_{\mathrm{k}}[\mathbf{u}_j \cdot s_{jj}] \right|^2 \\ &= \frac{1}{N} \cdot \sum_{i=1}^{N} \left| \sum_{n=1}^{N} u_{ni} \cdot \sigma_i \cdot e^{-j \cdot 2 \cdot \pi \cdot (k-1) \cdot (n-1)/N} \right|^2 \end{aligned} \tag{19}$$

where $\omega_k = \dfrac{k}{2 \cdot T_p \cdot N}$, T_p – sampling period, $\sigma_i = s_{ii}$ – i – singular value of $\mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}^{\mathrm{T}} = \hat{\mathbf{C}} \cdot \mathbf{L}^{\mathrm{F}} \cdot \hat{\mathbf{B}}$ decomposition.

5. AMPLITUDE AND PHASE CHARACTERISTICS APPROXIMATION

The relation between input and output power spectrum density and amplitude characteristics are described following.

$$\mathbf{S}_{\mathrm{y}}(\omega_k) = \left| \mathbf{G}(\omega_k) \right|^2 \cdot \mathbf{S}_{\mathrm{x}}(\omega_k) \tag{20}$$

taking into account theorem 5,

$$\left| \mathbf{G}(\omega_k) \right| = \sqrt{\mathbf{S}_{\mathrm{y}}(\omega_k)} \tag{21}$$

and finally

$$\begin{aligned} \left| \mathbf{G}(\omega_k) \right| &= \sqrt{\frac{1}{N} \cdot \sum_{j=1}^{N} \sigma_j^2 \cdot \left| \mathrm{DFT}_{\mathrm{k}}[\mathbf{u}_j] \right|^2} \\ &= \sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N} \left| \sigma_i \cdot \sum_{n=1}^{N} u_{ni} \cdot e^{-j \cdot 2 \cdot \pi \cdot (k-1) \cdot (n-1)/N} \right|^2} \end{aligned} \tag{22}$$

Phase characteristics, can be approximated by

$$\begin{aligned} \varphi(\omega_k) &= \arg\left(\sum_{j=1}^{N} \sigma_j \cdot \frac{\mathrm{DFT}_{\mathrm{k}}[\mathbf{u}_j]}{\mathrm{DFT}_{\mathrm{k}}[\mathbf{v}_j]} \right) \\ &= \arg\left(\sum_{i=1}^{N} \left(\sigma_i \cdot \frac{\sum_{n=1}^{N} u_{ni} \cdot e^{-j \cdot 2 \cdot \pi \cdot (k-1) \cdot (n-1)/N}}{\sum_{n=1}^{N} v_{ni} \cdot e^{-j \cdot 2 \cdot \pi \cdot (k-1) \cdot (n-1)/N}} \right) \right) \end{aligned} \tag{23}$$

Operator's notation, for which has been defined fundamental frequency analysis tools, is useful for description and simulation for time variant and time invariant systems and for a class of nonlinear systems all of these systems are defined on finite time horizon.

The analysis allow you to identify whether the system invariant is. If the system is variant the analysis shows how strong the varying is. For time-invariant systems power density spectrum of every corresponding column of matrices **U** and **V** is the same, for time varying systems is different. It is easy to account for. If the system is linear time-invariant, it will not introduce or modify normalized eigenvectors after passing the system, apart from shifting phase. It mean that a new frequencies will not be introduced into the spectrum. The degree of varying of the system can be defined as follows

$$S_{\mathrm{var}} = \sqrt{\frac{1}{N} \cdot \sum_{j=1}^{N} \left| \left| \mathrm{DFT}_{\mathrm{k}}[\mathbf{u}_j] \right|^2 - \left| \mathrm{DFT}_{\mathrm{k}}[\mathbf{v}_j] \right|^2 \right|_{(k)}} \tag{24}$$

For time invariant system value of the coefficient is equal to zero.

6. NUMERICAL EXAMPLES

Selected results of frequency characteristics approximation for time-varying systems using discrete operators, Singular Value Decomposition, and Discrete Fourier Transform for three different systems are presented below.

6.1. Oscillatory element with variable resonant frequency

The system is discretised analogue oscillatory, variable structure element. Oscillatory element is described by following differential equation

$$\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 2 \cdot \beta(t) \cdot \omega_0(t) \cdot \frac{\mathrm{d}y(t)}{\mathrm{d}t} + \omega_0^2(t) \cdot y(t) = k \cdot \omega_0^2(t) \cdot u(t),$$

corresponding transfer function in assumption, that $\omega_0(t) = \omega_0$, $\beta(t) = \beta$ are constant can be written as

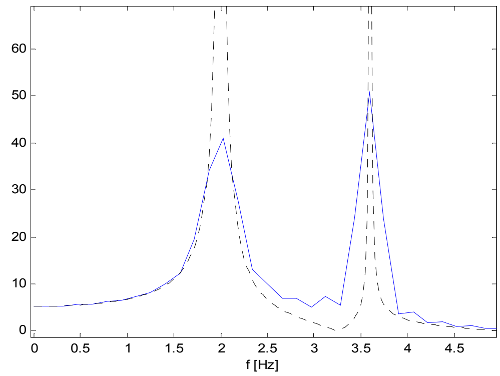
$$G_{osc}(s) = \frac{k \cdot \omega_0^2}{s^2 + 2 \cdot \beta \cdot \omega_0 \cdot s + \omega_0^2}.$$

Sampling period are equal to T=0.1s and simulation horizon N=64 (time horizon 6.4s, resolution in frequency domain Δf=0.156Hz).

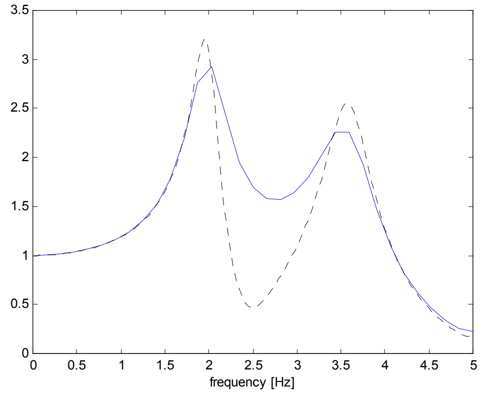
Resonant frequency before change ($t \le 3.2$s) is f_1=2Hz and after change ($t > 3.2$s) is f_2=3.6Hz.

Gain k=1 and damping factors are equal to $\beta_1 = 4 \cdot 10^{-4}$, $\beta_2 = 2.2 \cdot 10^{-4}$ (simulation on Figure 1a) and $\beta_1 = \beta_2 = 0.08 \cdot 10^{-4}$ (simulation on Figure 1b)

Time-invariant transfer function for average system is $G_i(z)=\frac{1}{2}\cdot\left(G_1(z)+G_2(z)\right)$ (G_1 and G_2 conversion have been done by zero order hold (ZOH) method).



(a)



(b)

Figure 1. Amplitude characteristics for damping factors $\beta_1=4\cdot10^{-4}$, $\beta_2=2.2\cdot10^{-4}$ (a) and $\beta_1=\beta_2=0.08\cdot10^{-4}$ (b), determined using SVD-DFT method for time-varying (solid line) and for average time-invariant transfer function $G_i(z)$ (dashed line).

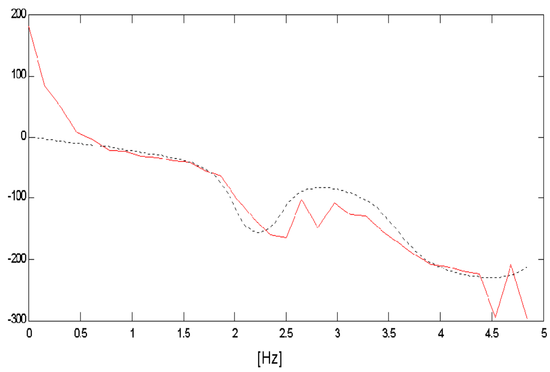


Figure 2. Phase characteristics for damping factors $\beta_1=4\cdot10^{-4}$, $\beta_2=2.2\cdot10^{-4}$, determined using SVD-DFT method for time-varying (solid line) and for average time-invariant transfer function $G_i(z)$ (dashed line).

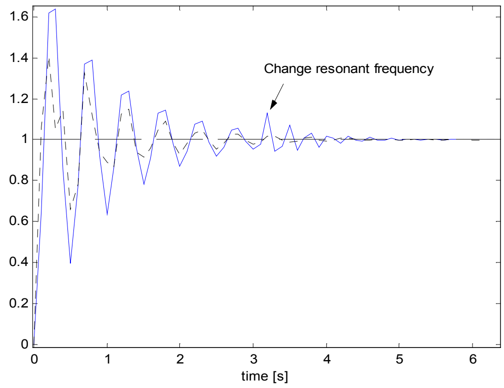


Figure 3. Step response for damping factors $\beta_1=\beta_2=0.08\cdot10^{-4}$, for time-varying system (solid line) and for average time-invariant transfer function $G_i(z)$ (dashed line).

Computed value of nonlinearity coefficient $S_{var}=1.5067$.

6.2. Low pass, third order, minimal phase digital, time invariant filter

The system is given by zeros-poles model. Sampling period is equal to $T_p=0.5s$. System zeros and poles are

$$\mathbf{z}=\begin{bmatrix}0.51+0.26i & 0.51-0.26i & 0.71\end{bmatrix}^T$$

$$\mathbf{p}=\begin{bmatrix}0.58+0.39i & 0.58-0.39i & 0.55 & 0.72\end{bmatrix}^T$$

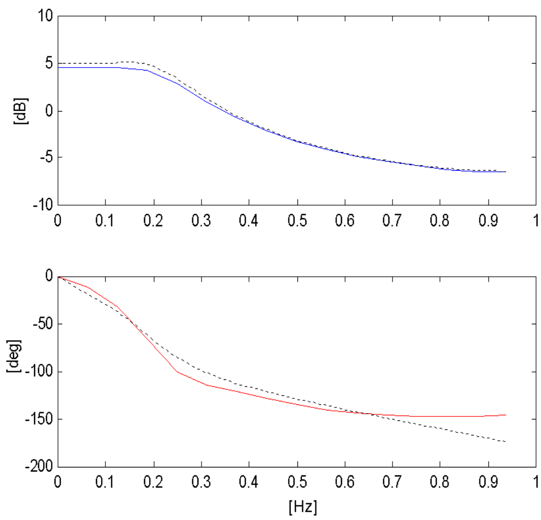


Figure 4. Bode characteristics, determined using SVD-DFT method (solid line) and using classical method (dashed line).

6.3. Variable structure, minimal phase, fourth order, time varying system

The system is described as zeros-poles model. Sampling period is equal to $T_p=0.1s$, time horizon is $N=32$ steps. The structure has been changed after N_p steps.

System before change

$$\mathbf{z} = \begin{bmatrix} 0.3403 + 0.0771i \\ 0.3403 - 0.0771i \\ -0.4317 \end{bmatrix} \quad \mathbf{p} = \begin{bmatrix} 0.1085 + 0.7459i \\ 0.1085 - 0.7459i \\ 0.2652 + 0.0924i \\ 0.2652 - 0.0924i \end{bmatrix}$$

System after N_p steps

$$\mathbf{z} = \begin{bmatrix} 0.3403 + 0.000771i \\ 0.3403 - 0.000771i \\ -0.4317 \end{bmatrix} \quad \mathbf{p} = \begin{bmatrix} 0.1085 + 0.007459i \\ 0.1085 - 0.007459i \\ 0.2652 + 0.000924i \\ 0.2652 - 0.000924i \end{bmatrix}$$

Computed value of variability coefficient S_{var}=0.9442 for N_p=3 and S_{var}=1.2009 for N_p=15.

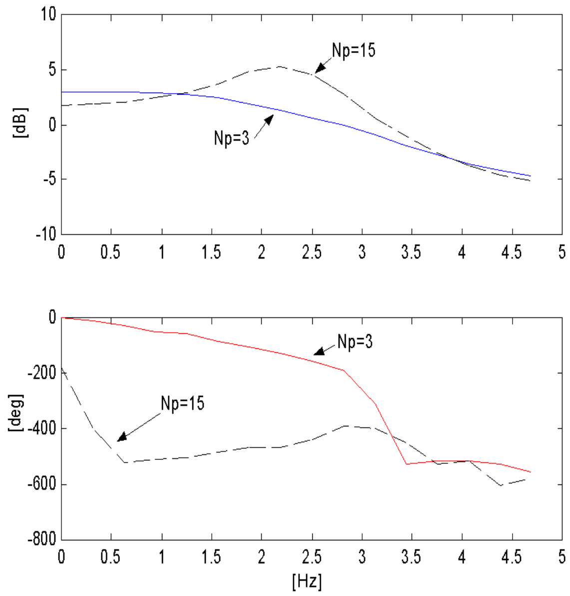


Figure 5. Bode characteristics, determined using SVD-DFT method for variable structure system. Solid line – change structure point N_p=3, and dashed line - change structure point N_p=15

7. CONCLUSION

Frequency analysis is well-known. Its definition is clear and the area of applications is wide. In real life, very often model of the system can be varied, but it is possible to neglect it in some cases. The method which was presented could be using either for time-invariant or time-varying systems, always on finite time horizon. A few numerical examples of time-invariant systems have been computed in section 6. The results are quite similar to the classical analysis. The main advantage of the *SVD-DFT* method is existence of extension for analysis for time-varying systems. It is difficult to compare results for time-varying systems, because classical Fourier Transform was not defined for such a systems. It is assumed, that one of another possible approximations could be weighted mean of transfer functions, which is used to compare with characteristics for time-varying systems with the new method. It should be clear, that the differences between characteristics are results of differences in computing. There do not exist reference characteristics for time-varying systems, defined on a finite time horizon, in fact. Another important thing is that, the method gives the information, wheather the system time-invariant, or time-varying is and about the strenght of this varying.

Newertheless, the results for practical applications look optimistically, for these methods. In spite of many advantages of the method, there exist a few weak points, which were appeared during simulations. There are a wide area of applications, for example: systems analysis, time-varying systems estimates, e.g. non-stationarity and non-linearity degree determining (relative and absolute). Applications in fast, real-time damage detection look very interesting.

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