COMPARATIVE BOUNDARY AND SENSITIVITY ANALYSIS FOR UNCERTAIN DYNAMICAL SYSTEMS

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Abstract. The paper develops analysis for models of uncertainty for dynamical, discrete-time control systems on finite time horizon. Various models of uncertainty are analysed: additive, subtractive, and multiplicative. An analysis for electrical circuit with real, perturbed parameters is carried out. It is assumed, that the uncertain parameters are bounded, and could be described by rectangular distribution. A few cases of systems are considered. In the first case system is time-invariant. In the second case, system is time-variant. Output errors are estimated using discrete evolution operators. As a comparison sensitivity analysis for time-invariant sis has been carried out. The results from estimations are compared to the set of the worst-case uncertain parameters related to output error.

Keywords. Disturbed Systems, Estimation, Sampled Data Systems, State Space Model.

1. INTRODUCTION

Uncertainty can be introduced into model of the system in many ways. Most often the method depends on used description. It is possible to describe the system by difference or differential equations, recurrent state space model or using operators. The most known is the Laplace's operator, which gives well known transfer function, but also it is possible to use evolutionary operators defined in (Emirsajłow 1999; Orłowski 2000/1-2, 2001) or operators in general.

Apart from the system description it is possible to distinguish six structures that allow introducing uncertainty into the system.

2. STRUCTURES OF UNCERTAINTIES

Six uncertain model's structures are presented below; **G** can be a matrix, transfer function or system's operator. Perturbation Δ can be a scalar, matrix, transfer function or operator. The most popular are scalars and matrices. All structures can be used both for linear time-invariant and time-varying or non-linear systems.

2.1. Additive errors

The typical uncertainties for additive errors are additive plant errors, uncertain right half plane zeros (Hoary 1996). Input responses to output commands are the typical performance specification. Block diagram for systems with additive errors is drawn on Fig. 1. Mathematical description (1) and conditions for invertibility for a matrix, (2) for ($\mathbf{I}+\mathbf{G}_\Delta$) and (3) for $\mathbf{G}_\Delta$ are following

$$\mathbf{G}_\Delta = \mathbf{G} + \Delta \quad (1)$$

$$\sigma_{\max}(\Delta) < \sigma_{\min}(\mathbf{I}+\mathbf{G}) \ (2), \quad \sigma_{\max}(\Delta) < \sigma_{\min}(\mathbf{G}) \quad (3)$$

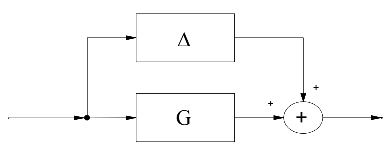

Fig. 1. Additive perturbation.

2.2. Subtractive errors

The typical uncertainties for subtractive are low frequency plant parameter errors; changing number of right half plane poles. Output errors to input commands and disturbances are the typical performance specification. Block diagram for systems with subtractive errors is drawn on Fig. 2. Mathematical description and conditions for invertibility are as follow ($\mathbf{I}+\mathbf{G}_\Delta$) (5) and $\mathbf{G}_\Delta$ (6).

$$\mathbf{G}_\Delta = (\mathbf{G}^{-1} + \Delta)^{-1} \qquad (4)$$

$$\sigma_{\max}(\Delta) < \sigma_{\min}(\mathbf{I} + \mathbf{G}^{-1}) \;\; (5),\; \sigma_{\max}(\Delta) < \sigma_{\min}(\mathbf{G}^{-1}) \;\; (6)$$

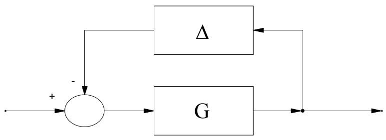

Fig. 2. Subtractive perturbation.

2.3. Post-Multiplicative errors

The typical uncertainties for post-multiplicative errors are output errors, neglected high frequency dynamics; changing number of right half plane zeros. Sensor noise attenuation, output responses to output commands are the typical performance specification. Block diagram for systems with post-multiplicative errors is drawn on Fig. 3. Mathematical description is

$$\mathbf{G}_\Delta = \mathbf{G} \cdot (\mathbf{I} + \Delta) \qquad (7)$$

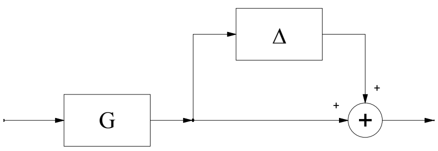

Fig. 3. Post-Multiplicative perturbation.

2.4. Pre-Multiplicative errors

The typical uncertainties for pre-multiplicative errors are input errors, neglected high frequency dynamics, changing number of right half plane zeros. Input responses to input commands are the typical performance specification. Block diagram for systems with pre-multiplicative errors is drawn on Fig. 4. Mathematical description is

$$\mathbf{G}_\Delta = (\mathbf{I} + \Delta) \cdot \mathbf{G} \qquad (8)$$

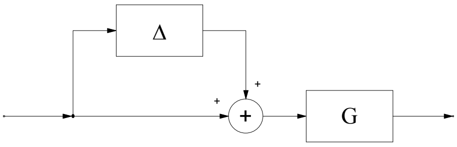

Fig. 4. Pre-Multiplicative perturbation.

Conditions for invertibility are the same for both post and pre-multiplicative errors ($\mathbf{I}+\mathbf{G}_\Delta$) (9) and $\mathbf{G}_\Delta$ (10).

$$\sigma_{\max}(\Delta) < \sigma_{\min}(\mathbf{I} + \mathbf{G}^{-1}) \;\; (9),\;\; \sigma_{\max}(\Delta) < 1 \quad (10)$$

2.5. Post-Divisional errors

The typical uncertainties are low frequency plant parameter errors, changing number of right half plane poles. Output sensitivity, output errors to output command disturbances are the typical performance specification. Block diagram for systems with post-divisional errors is drawn on Fig. 5. The system can be described by

$$\mathbf{G}_\Delta = \mathbf{G} \cdot (\mathbf{I} + \Delta)^{-1} \qquad (11)$$

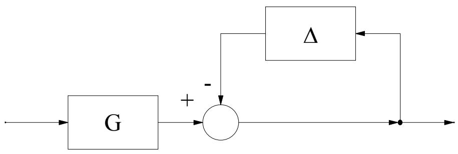

Fig. 5. Post-Divisional perturbation.

2.6. Pre-Divisional errors

The typical uncertainties are low frequency plant parameter errors, changing number of right half plane poles. Input sensitivity and input errors to input command disturbances are the typical performance specification. Block diagram for systems with pre-divisional errors is drawn on Fig. 6. The system can be described by

$$\mathbf{G}_\Delta = (\mathbf{I} + \Delta)^{-1} \cdot \mathbf{G} \qquad (12)$$

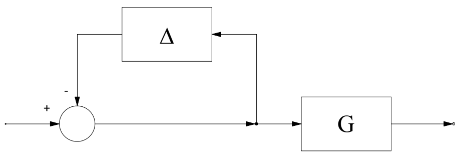

Fig. 6. Pre-Divisional perturbation.

Conditions for invertibility are the same for both structures ($\mathbf{I}+\mathbf{G}_\Delta$) (13) and $\mathbf{G}_\Delta$ (14) can be described by $\sigma_{\max}(\Delta) < \sigma_{\min}(\mathbf{I} + \mathbf{G})$ (13), $\sigma_{\max}(\Delta) < 1$ (14)

3. ANALYSIS OF AN ELECTRICAL CIRCUIT WITH UNCERTAINTIES

For given system, some aspects determine the model of uncertainty. First is modelling. Some structures are more suitable, and the model is clearer. Also it is easier to understand, how system works. Second is computational. Some tools are designed for using with a certain model and cannot be use with others. Fig. 7 shows electrical circuit, in which some parameters are uncertain.

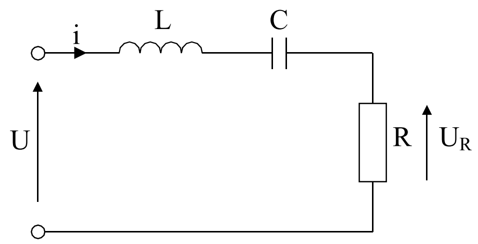

Fig. 7. Electrical circuit RLC.

Continuous-time state-space model is

$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{dt}} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L\cdot C\cdot 1000} \\ 1000 & 0 \end{bmatrix}\cdot\mathbf{x}(t) + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}\cdot u(t) \tag{15}$$
$$u_R(t) = \begin{bmatrix} R & 0 \end{bmatrix}\cdot\mathbf{x}(t)$$

where $\mathbf{x}(t) = \begin{bmatrix} i(t) & 1000\cdot\int_0^t i\cdot dt \end{bmatrix}^{\mathrm{T}}$.

The state variables have been chosen to minimize the norm of system matrix **A**. For state variables given above the norm is $\|\mathbf{A}\| = 1051$. If the state variables were following $\mathbf{x}(t) = \begin{bmatrix} i(t) & \int_0^t i\cdot dt \end{bmatrix}^{\mathrm{T}}$, the norm would be $\|\mathbf{A}\| = 10^6$. Minimisation of norm of matrix **A** is required for the method given by equations (17-19).

Corresponding discrete-time state-space model is

$$\mathbf{x}(k+1) = \begin{bmatrix} 1-\frac{R}{L}\cdot T & -\frac{T}{L\cdot C\cdot 1000} \\ T\cdot 1000 & 1 \end{bmatrix}\cdot\mathbf{x}(k) + \begin{bmatrix} \frac{T}{L} \\ 0 \end{bmatrix}\cdot u(k) \tag{16}$$
$$u_R(k) = \begin{bmatrix} R & 0 \end{bmatrix}\cdot\mathbf{x}(k)$$

Values in simulation: $T = 5\cdot 10^{-5}\,\mathrm{s}$ and time horizon, N=50, $k = 0,1,\ldots,N-1$.

Estimates of output error have been calculated using theorems from (Emirsajłow 1999; Orłowski 2000/1, 2001). The main results are

$$\|\mathbf{y}_\Delta(\cdot)-\mathbf{y}_p(\cdot)\| \le \left[\ \|\hat{\mathbf{C}}\cdot\mathbf{L}^{\mathbf{F}}\|\cdot\delta_{\mathrm{A}}+\delta_{\mathrm{C}}\right] \cdot\frac{\|\mathbf{x}_p\|+\|\mathbf{L}^{\mathbf{F}}\|\cdot\delta_{\mathbf{B}}\cdot\|\mathbf{v}_p\|}{1-\|\mathbf{L}^{\mathbf{F}}\|\cdot\delta_{\mathbf{A}}} + \|\hat{\mathbf{C}}\cdot\mathbf{L}^{\mathbf{F}}\|\cdot\delta_{\mathrm{B}}\cdot\|\mathbf{v}_p\| \tag{17}$$

$$\|\mathbf{y}_\Delta(N)-\mathbf{y}_p(N)\| \le \frac{\|\hat{\mathbf{C}}\cdot\mathbf{K}^{\mathbf{F}}\|\cdot\delta_{\mathrm{A}}+\delta_{\mathrm{CN}}}{1-\|\mathbf{L}^{\mathbf{F}}\|\cdot\delta_{\mathrm{A}}} \cdot\left[\ \|\mathbf{x}_p\|+\|\mathbf{L}^{\mathbf{F}}\|\cdot\delta_{\mathrm{B}}\cdot\|\mathbf{v}_p\|\right]+\|\hat{\mathbf{C}}\cdot\mathbf{K}^{\mathbf{F}}\|\cdot\delta_{\mathrm{B}}\cdot\|\mathbf{v}_p\| \tag{18}$$

under condition

$$\delta_{\mathrm{A}} < \|\mathbf{L}^{\mathrm{F}}\|^{-1} \tag{19}$$

Equation (17) is the estimate of trajectory error norm. Moreover equation (18) denotes norm of terminal output error.

Operator $\mathbf{L}^{\mathrm{F}} \in \mathcal{L}((\mathbf{R}^n)^N,(\mathbf{R}^n)^N)$ is defined for k=2,3,..,N and $\mathbf{h}(i)\in\mathcal{L}(\mathbf{R}^n)$ as evolutionary by

$$(\mathbf{L}^{\mathbf{F}}\mathbf{h})(k) = \sum_{i=0}^{k-2}\left[\prod_{j=i+1}^{k-1}\mathbf{A}(j)\right]\cdot\mathbf{h}(i)+\mathbf{h}(k-1) \tag{20}$$

or equivalently by block matrix operator

$$\mathbf{L}^{\mathrm{F}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{L}^{\mathrm{P}}(1,1) & \mathbf{I} & \mathbf{0} & \vdots & \vdots \\ \vdots & \ddots & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{L}^{\mathrm{P}}(1,N-2) & \cdots & \mathbf{L}^{\mathrm{P}}(N-2,N-2) & \mathbf{I} & \mathbf{0} \end{bmatrix} \tag{21}$$

where

$$\mathbf{L}^{\mathrm{P}}(i,k) = \prod_{j=i}^{k}\mathbf{A}(j) \tag{22}$$

Operator $\mathbf{K}^{\mathbf{F}}$ is a block vector of the last row of matrix (21). Equivalently $\mathbf{K}^{\mathbf{F}}$ can be computed by substituting k=N in equation (20). Using matrix notation, operator $\hat{\mathbf{C}}$ is defined as follows

$$\hat{\mathbf{C}} = \begin{bmatrix} \mathbf{C}(0) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}(N-1) \end{bmatrix} \tag{23}$$

There are two different ways for computing the norms of operators $\mathbf{L}^{\mathbf{F}}$, $\hat{\mathbf{C}}\mathbf{L}^{\mathbf{F}}$, $\mathbf{C}\mathbf{K}^{\mathbf{F}}$. First is based on discrete difference Riccati equation and was presented in (Emirsajłow 1999; Orłowski 2000/1). Second is based on singular value decomposition (Orłowski 2001) and other matrix tools. All operators' norms in this paper have been computed using matrix notation and matrix tools.

3.1. Uncorrelated perturbations of linear time-invariant system

Parameters of the system are as follows:

$$C = (1+\delta_C)\cdot C_n = (1+\delta_C)\cdot 1\mu\mathrm{F},$$
$$\delta_C \in \langle -0.01, 0.01\rangle,\ C\in\langle 0.99, 1.01\rangle\,\mu\mathrm{F} \tag{24}$$

$$L = (1+\delta_L)\cdot L_n = (1+\delta_L)\cdot 1\mathrm{H},$$
$$\delta_L \in \langle -0.03, 0.03\rangle,\ L\in\langle 0.97, 1.03\rangle\,\mathrm{H} \tag{25}$$

$$R = (1+\delta_R)\cdot R_n = (1+\delta_R)\cdot 100\Omega\,,$$
$$\delta_{R2} \in \langle -0.1, 0.1\rangle\,,\ R \in \langle 90, 110\rangle \Omega \qquad (26)$$

Coefficients of additive perturbations and their upper and lower bounds can be calculated as follow

$$a_{11} = a_{11n} + a_{11\delta} \quad (27), \qquad a_{11\delta} \in \langle a_{11\delta-}, a_{11\delta+}\rangle \quad (28)$$

$$a_{11\delta-} = \left(\frac{R_n}{L_n} - \frac{R_+}{L_-}\right)\cdot T\,,\ a_{11\delta+} = \left(\frac{R_n}{L_n} - \frac{R_-}{L_+}\right)\cdot T \quad (29)$$

$$a_{12\delta-} = \left(\frac{1}{L_n\cdot C_n} - \frac{1}{L_-\cdot C_-}\right)\cdot\frac{T}{1000}\,, \qquad (30)$$

$$a_{12\delta+} = \left(\frac{1}{L_n\cdot C_n} - \frac{1}{L_+\cdot C_+}\right)\cdot\frac{T}{1000} \qquad (31)$$

$$b_{1\delta-} = \left(\frac{1}{L_+} - \frac{1}{L_n}\right)\cdot T\,,\ b_{1\delta+} = \left(\frac{1}{L_-} - \frac{1}{L_n}\right)\cdot T \quad (32)$$

$$c_{1\delta-} = R_- - R_n\,, \quad c_{1\delta+} = R_+ - R_n \qquad (33)$$

The discretised model of the system can be rewritten in following form

$$\mathbf{x}(k+1) = \left(\begin{bmatrix} 1-\frac{R}{L}\cdot T & -\frac{T}{L\cdot C\cdot 1000} \\ T\cdot 1000 & 1 \end{bmatrix} + \Delta_{\mathbf{A}}\right)\cdot\mathbf{x}(k) + \left(\begin{bmatrix} \frac{T}{L} \\ 0 \end{bmatrix} + \Delta_{\mathbf{B}}\right)\cdot u(k)$$
$$u_R(k) = \left(\begin{bmatrix} R & 0 \end{bmatrix} + \Delta_{\mathbf{C}}\right)\cdot\mathbf{x}(k)$$
$$(34)$$

$$\Delta_{\mathbf{A}} = \begin{bmatrix} a_{11\delta} & a_{12\delta} \\ 0 & 0 \end{bmatrix},\ \Delta_{\mathbf{B}} = \begin{bmatrix} b_{1\delta} \\ 0 \end{bmatrix},\ \Delta_{\mathbf{C}} = \begin{bmatrix} c_{1\delta} & 0 \end{bmatrix} (35)$$

Using the technique described in (Orłowski 2001), the operator's norms $\|\cdot\|_2$, $\|\cdot\|_\infty$ have been calculated and collected in Table 1. Then using the tools for analysis uncertain systems, estimates of $\|\mathbf{y}_\Delta(\cdot) - \mathbf{y}_p(\cdot)\|_2$ (dotted line) and $\|\mathbf{y}_\Delta(N) - \mathbf{y}_p(N)\|_2$ (triangle) have been obtained and drawn on Fig. 8.

The real error's norm characteristics $\|\mathbf{y}_\Delta(\cdot) - \mathbf{y}_p(\cdot)\|_2$ have been obtained for extreme positive and negative (solid line) perturbation's matrices.

Table 1. Operators' norms $\|\cdot\|_2$ and $\|\cdot\|_\infty$.

Operator \ Norm	$\|\cdot\|_2$	$\|\cdot\|_\infty$
$\mathbf{L^F}$	31.6198	62.9636
$\mathbf{N^F}$	7.0923	1.4112
$\hat{\mathbf{C}}$	100	100
$\hat{\mathbf{B}}$	3.0000e-005	3.0000e-005
$\mathbf{K^F}$	7.0226	62.9636
$\mathbf{C}\cdot\mathbf{L^F}$	2.9062e+003	6.0107e+003
$\mathbf{C}\cdot\mathbf{N^F}$	613.8774	134.4651
$\mathbf{C}\cdot\mathbf{K^F}$	671.9687	6.0107e+003

Calculated norms have been collected in Table 2. Fig. 9 shows output trajectory estimates $\mathbf{y}_p \pm \|\mathbf{y}_\Delta - \mathbf{y}_p\|_\infty$ (dotted line) and terminal output vector (triangles). Trajectories obtained for real extreme matrices perturbations are drawn solid line.

Table 2. Estimates and extreme norms $\|\cdot\|_2$ and $\|\cdot\|_\infty$.

Norm	Estimate	Extreme posit. value	Extreme negat. value	Relative error
$\|\mathbf{y}_\Delta(\cdot) - \mathbf{y}_p(\cdot)\|_2$	0.1031	0.0702	0.0651	0.4696
$\|\mathbf{y}_\Delta(N) - \mathbf{y}_p(N)\|_2$	0.0237	0.0154	0.0141	0.5460
$\|\mathbf{y}_\Delta(\cdot) - \mathbf{y}_p(\cdot)\|_\infty$	0.0279	0.0154	0.0141	0.8153
$\|\mathbf{y}_\Delta(N) - \mathbf{y}_p(N)\|_\infty$	0.0279	0.0154	0.0141	0.8153

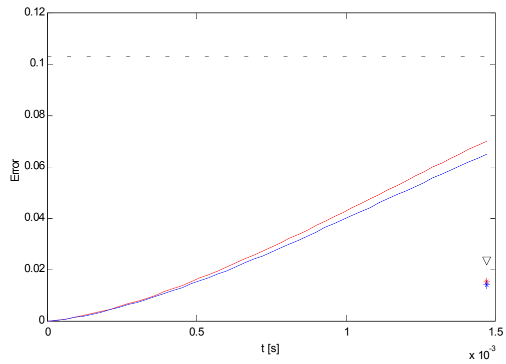

Fig. 8. Errors norms $\|\mathbf{y}_\Delta(\cdot) - \mathbf{y}_p(\cdot)\|_2$ (solid) and $\|\mathbf{y}_\Delta(N) - \mathbf{y}_p(N)\|_2$ (stars) characteristics in function of time and their upper bounds (dotted, triangle).

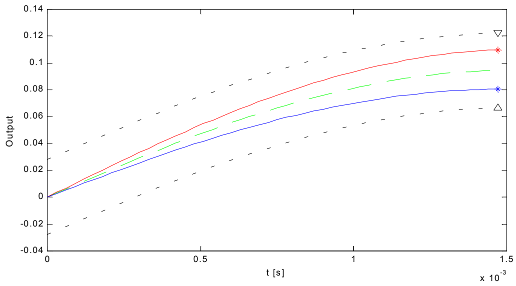

Fig. 9. Output trajectories $\mathbf{y}_\Delta(\cdot)$ (solid), $\mathbf{y}_p(\cdot)$ (dashed) in dependence of time, $\mathbf{y}_\Delta(N)$ (stars) and their upper estimates $\mathbf{y}_p \pm \|\mathbf{y}_\Delta - \mathbf{y}_p\|_\infty$ (dotted, triangles).

3.2. Sensitivity analysis for time-invariant system

Sensitivity analysis is very well known tool for analysis of uncertain control systems in time-domain. Comparative analysis for system described in section 3.1 using sensitivity analyses has been carried out. Function of output error follows from total differential and the main formula is

$$\Delta y(\cdot) \approx \left|\frac{\partial y(\cdot)}{\partial R}\right| \cdot \Delta R + \left|\frac{\partial y(\cdot)}{\partial L}\right| \cdot \Delta L + \left|\frac{\partial y(\cdot)}{\partial C}\right| \cdot \Delta C \quad (36)$$

Estimated output of uncertain system is equal to

$$y_\Delta(\cdot) = y(\cdot) \pm \Delta y(\cdot) \quad (37)$$

Numerical data has been calculated using Matlab. Fig. 10, 11 show output trajectory for nominal (unperturbed) system (dashed), upper and lower bounds of sensitivity estimates (dotted) and trajectories of systems with extreme parameters' values (solid). Fig. 10 shows first 1.5ms likewise Fig. 9. Fig. 11 shows the same characteristics plotted for longer (80ms) time.

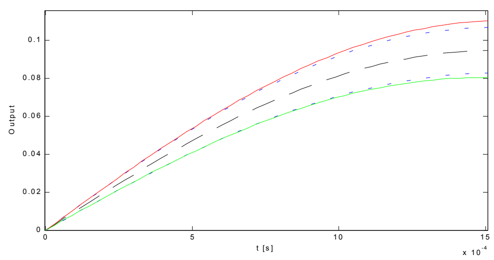

Fig. 10. Output trajectories and estimates for uncertain system (1.5ms).

Estimates obtained from sensitivity analysis are nearby system with extreme parameters. Sensitivity analysis does not guarantee extreme bounds for system. Analysis of system using (17-19) always gives guaranteed extreme bounds. Disadvantage of the method is more conservative estimate.

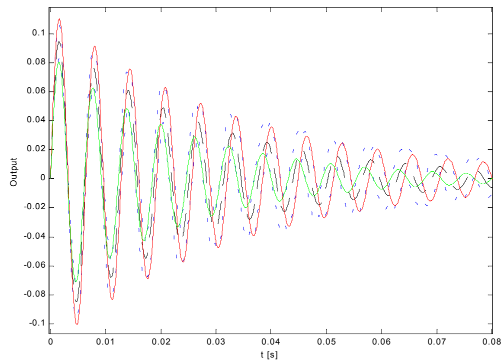

Fig. 11. Output trajectories and estimates for uncertain system (80ms).

3.3. Uncorrelated perturbations of linear time-varying system

Now it is assumed, that the system is time varying. Parameters C and L are the same as in previous example. Parameter R is defined as follow

$$R_n(k) = \left(1 + \frac{k}{N}\right) \cdot 100\Omega \quad (38)$$

$$R(k) = \left\langle R_n(k) - 10\Omega, R_n(k) + 10\Omega \right\rangle \quad (39)$$

hence $R_-(k) = R_n(k) - 10\Omega$, $R_+(k) = R_n(k) + 10\Omega$

Calculated norms have been collected in Tab. 3. Fig. 12 shows output trajectory estimates $\mathbf{y}_p \pm \|\mathbf{y}_\Delta - \mathbf{y}_p\|_\infty$ (dotted line) and terminal output vector (triangles). Trajectories obtained for real extreme matrices perturbations are drawn solid line. Estimates of upper bounds output trajectories and output two signals are drawn on Fig. 12.

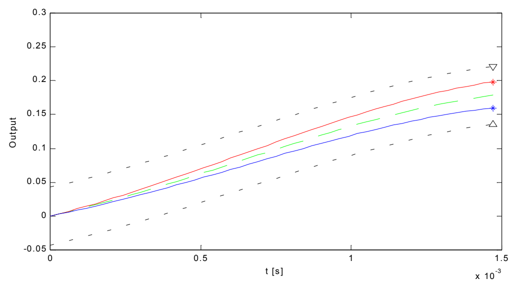

Fig. 12. Output trajectories $\mathbf{y}_\Delta(\cdot)$ (solid), $\mathbf{y}_p(\cdot)$ (dashed) in dependence of time, $\mathbf{y}_\Delta(N)$ (stars) and their upper estimates $\mathbf{y}_p \pm \|\mathbf{y}_\Delta - \mathbf{y}_p\|_\infty$ (dotted, triangles).

Using the technique described in (Orłowski 2001), the operator's norms $\|\cdot\|_2$, $\|\cdot\|_\infty$ have been calculated. Then using the tools for analysis uncertain systems, estimates of $\left\|\mathbf{y}_\Delta(\cdot)-\mathbf{y}_p(\cdot)\right\|_2$ (dotted line) and $\left\|\mathbf{y}_\Delta(N)-\mathbf{y}_p(N)\right\|_2$ (triangle) have been obtained and drawn on Fig. 13. The real error's norm characteristics $\left\|\mathbf{y}_\Delta(\cdot)-\mathbf{y}_p(\cdot)\right\|_2$ have been obtained for extreme positive and negative (solid line) perturbation's matrices.

Table 3. Estimates and extreme norms $\|\cdot\|_2$ and $\|\cdot\|_\infty$.

Norm	Estimate	Extreme posit. value	Extreme negat. value	Relative error
$\left\|\mathbf{y}_\Delta(\cdot)-\mathbf{y}_p(\cdot)\right\|_2$	0.1305	0.0850	0.0792	0.5342
$\left\|\mathbf{y}_\Delta(N)-\mathbf{y}_p(N)\right\|_2$	0.0321	0.0200	0.0186	0.6037
$\left\|\mathbf{y}_\Delta(\cdot)-\mathbf{y}_p(\cdot)\right\|_\infty$	0.0427	0.0200	0.0186	1.1303
$\left\|\mathbf{y}_\Delta(N)-\mathbf{y}_p(N)\right\|_\infty$	0.0427	0.0200	0.0186	1.1301

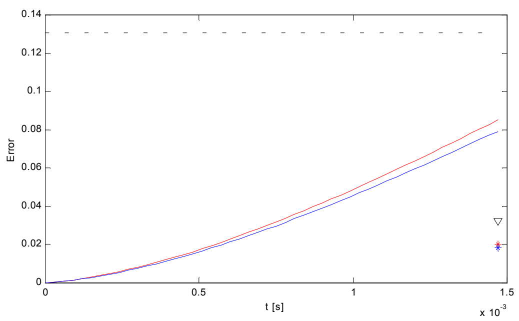

Fig. 13. Errors norms $\left\|\mathbf{y}_\Delta(\cdot)-\mathbf{y}_p(\cdot)\right\|_2$ (solid) and $\left\|\mathbf{y}_\Delta(N)-\mathbf{y}_p(N)\right\|_2$ (stars) characteristics in function of time and their upper bounds (dotted, triangle).

4. CONCLUSION

The analysis of the uncertain system and the deviations' estimates are accomplished in time domain and in finite time horizon. For time invariant and periodically varying systems, the operators are invariant and could be evaluated only once. The estimates computed in H_∞ space are more conservative then their equivalents in H_2. Nevertheless, when the process and measurement noises are normal and not negligible, it is easier to estimate the energy or power of the noises than the peak noise value.

The estimates' quality depends on time horizon and properties of the model. When the time horizon is shorter, the estimate is less consrvative and vice versa. On the other hand long time horizon is better , when there are considered effects of noises.

The developed estimates can be used also in various control desgn tasks for perturbed non-stationary linear discrete time systems. Proposed method does not require detailed knowledge about the system. It is enough to know only coefficients of the model, and their deviations' estimates. The coefficients can be taken, for example by model identification.

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